

# APPARENT MAGNITUDES IN AN INHOMOGENEOUS UNIVERSE: THE GLOBAL VIEWPOINT

HAMISH G. ROSE

University of Canterbury, Christchurch, New Zealand  
 h.rose@phys.canterbury.ac.nz  
 Draft version February 1, 2008

## ABSTRACT

Apparent magnitudes are important for high precision cosmology. It is generally accepted that weak gravitational lensing does not affect the relationship between apparent magnitude and redshift. By considering metric perturbations it is shown that objects observed in an inhomogeneous universe have, on average, higher apparent magnitudes than those observed at the same redshift in a homogeneous universe.

*Subject headings:* gravitational lensing — galaxies: distances and redshifts — cosmology: observations — distance scale

## 1. INTRODUCTION

Implicit in many modern cosmological experiments, such as the Supernova Cosmology Project (Perlmutter et al. 1998) and the High-*z* Supernova Search (Schmidt et al. 1998), is the assumption that the relationship between the apparent magnitude of a distant object and the redshift of that object is unaffected by gravitational lensing.

Weinberg (1976) presents an argument that gravitational lensing does not, on average, affect the apparent magnitude – radial Robertson Walker coordinate relationship. His argument was based on photon number conservation and has been widely accepted, although recently it has come under closer scrutiny. Ellis et al. (1998) have reanalysed Weinberg’s argument and concluded that it is not valid if caustics are present, which only occurs with strong fields. Claudel (2000) considers Newtonian perturbations in the weak field limit and finds that, to first order in  $\kappa = 8\pi G/c^2$ , there is no deviation from the Friedmann Robertson Walker (FRW) result.

Weinberg’s original argument is presented in Section 2. In Section 3 I demonstrate using metric perturbations that the presence of inhomogeneities in the universe do, on average, affect the apparent magnitude – redshift relationship.

## 2. PHOTON NUMBER COUNT VERSUS COORDINATE DISTANCE

Consider an exactly FRW universe containing a source at (comoving coordinate)  $r = 0$  that emits  $N$  photons (due to, say, a cataclysmic event). Drawing a sphere around the source at  $r = r_{\text{obs}}$  (with surface area  $4\pi a_{\text{obs}}^2 r_{\text{obs}}^2$  where  $a(t)$  is the cosmological scale factor), an astronomer at  $r = r_{\text{obs}}$  with a telescope of area  $A$  will observe  $n$  photons satisfying

$$\frac{4\pi a_{\text{obs}}^2 r_{\text{obs}}^2}{A} = \frac{N}{n}. \quad (1)$$

Now consider the above situation with the matter inside the sphere distributed unevenly, thus lensing the photons. The number of photons observed by the astronomer may be different when compared to the previous situation. However, the total number of photons passing through the sphere is unchanged. If the area of the sphere at  $r = r_{\text{obs}}$  has not changed then any increase or decrease in photons seen by the astronomer must be compensated by a decrease or increase respectively in the number of photons observed by other astronomers. Furthermore, if there are a large number of astronomers at different points

on the sphere at  $r = r_{\text{obs}}$  then the average number of photons they observe must be distributed about  $n$  with a standard deviation that approaches 0 as their combined telescopes cover the sphere.

Weinberg’s argument is based on the assumption that the area of the sphere centred on  $r = 0$  and with radius  $r = r_{\text{obs}}$  is not affected by the mass distribution. The area of the sphere depends not only on  $r$  but also on  $a(t)$ . If inhomogeneities affect the time photons take to travel from  $r = 0$  to  $r = r_{\text{obs}}$  then inhomogeneities must also affect the area of the sphere. If this is the case then photon conservation does not imply that the observed apparent magnitude relationship is identical to the FRW apparent magnitude relationship. In Section 3 this is shown to be the case.

## 3. APPARENT MAGNITUDE VERSUS REDSHIFT

Consider a perturbed FRW dust universe with line element

$$ds^2 = c^2 dt^2 - a(t)^2 (1 - h(r, t))^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \right) \quad (2)$$

and an energy momentum tensor

$$T = \begin{bmatrix} \rho_b(1 + \delta(r, t)) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (3)$$

where  $\rho_b$  is the matter density in the FRW universe and  $\delta(r, t)$  describes the departure from homogeneity. The coordinates are comoving and peculiar motions are neglected so  $T$  contains no terms dependent on the velocity of matter.

The metric has determinant

$$\sqrt{-g} = a^3 (1 - h)^3. \quad (4)$$

Spherical symmetry has been retained so that any astronomer at  $r = r_{\text{obs}}$  makes the same observations of a source at  $r = 0$  (as in Section 2) as any other astronomer at  $r = r_{\text{obs}}$ . Spherical symmetry is an unnatural condition to impose upon inhomogeneities. However, no use is made here of the matter distribution except in that no observer is in a special location. In particular no dynamics are considered so spherical symmetry is a reasonable condition to impose.

Conditions are imposed upon the perturbed universe to ensure that it does not depart too far from FRW. Specifically, the

total mass content is the same as in a FRW universe, which means that the function  $a(t)$  is the same in both cases; and the inhomogeneities are small in both amplitude ( $|\delta| \ll 1$ ) and length. In every region small enough that the scale factor,  $a(t)$ , changes little in the time taken for the photon to travel through it,  $\delta$  averages to 0. Thus we impose the condition

$$\int_{\lambda} \frac{\delta(r(\lambda), t(\lambda))}{a(t(\lambda))} d\lambda = 0 \quad (5)$$

where  $\lambda$  is any parameterization along the geodesic. Finally,  $h(r, t) = 0$  at both the source and the observer's locations since the only effect under consideration here is due to matter inhomogeneities between the source and the observer.

Following Peebles (1993, page 276) (but note the different definition of  $h$  which allows the calculation to be carried out to all orders), the stress energy conservation law leads to

$$\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} T_0^{\mu}) = \frac{1}{2} g_{\mu\nu,0} T^{\mu\nu} = 0 \quad (6)$$

or

$$\frac{\dot{\rho}_b}{\rho_b} + \frac{\dot{\delta}}{1+\delta} = -\frac{3\dot{a}}{a} + \frac{3\dot{h}}{1-h}. \quad (7)$$

Since  $\dot{\rho}_b/\rho_b = -3\dot{a}/a$  in the unperturbed universe,

$$\frac{\dot{\delta}}{1+\delta} = \frac{3\dot{h}}{1-h} \quad (8)$$

which has solution

$$1-h = (1+\delta)^{-\frac{1}{3}}. \quad (9)$$

The integration constant has been determined by requiring that when  $\delta = 0$ ,  $h = 0$ .

By integrating along a radial null geodesic from emission at  $(t, r) = (t_{\text{em}}, 0)$  to observation at  $(t, r) = (t_{\text{obs}}, r)$ , the radial coordinate where a photon arrives at the observer in the perturbed universe can be determined and compared to the radial coordinate of the observer in the FRW universe.

In the perturbed FRW universe

$$ds^2 = 0 \Rightarrow cdt = a(t)(1-h(r, t)) \frac{dr}{\sqrt{1-kr^2}}. \quad (10)$$

Equation (10) gives a differential equation which may be solved for  $r(t)$ . The position of the photon is completely described by the function  $r(t)$ .  $h(t)$  is now defined along the geodesic as  $h(t) \equiv h(r(t), t)$ . Similarly,  $\delta(t) \equiv \delta(r(t), t)$ . Rearranging equation (10) and integrating, one obtains

$$\int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{cdt}{a(t)(1-h(t))} = \int_0^{r_{\text{obs}}} \frac{dr}{\sqrt{1-kr^2}}, \quad (11)$$

Similarly, in the FRW universe

$$\int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{cdt}{a(t)} = \int_0^{r_{\text{FRW}}} \frac{dr}{\sqrt{1-kr^2}}. \quad (12)$$

Substituting for  $1-h$  from equation (9) and making the second order approximation  $(1+\delta)^{1/3} \approx 1+\delta/3-\delta^2/9$ , equation (11) becomes

$$\int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{cdt}{a(t)} + \frac{1}{3} \int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{c\delta(t)dt}{a(t)} - \frac{1}{9} \int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{c\delta(t)^2 dt}{a(t)} = \int_0^{r_{\text{obs}}} \frac{dr}{\sqrt{1-kr^2}}. \quad (13)$$

Equations (12) and (13) are both written using comoving coordinates and proper time and so may be compared directly. The first term in equation (13) may be replaced using equation (12) and the second term is 0 due to equation (5) leaving

$$\frac{1}{9} \int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{c\delta(t)^2 dt}{a(t)} = \int_{r_{\text{obs}}}^{r_{\text{FRW}}} \frac{dr}{\sqrt{1-kr^2}}. \quad (14)$$

Since  $\delta^2 \geq 0$  and  $a > 0$  then  $r_{\text{FRW}} \geq r_{\text{obs}}$  and  $r_{\text{obs}} = r_{\text{FRW}}$  only if  $\delta = 0$  at all points along the lightcone. We may use this to compare the observed number of photons in the perturbed universe and the FRW universe. From equation (1),

$$\frac{n_{\text{obs}}}{n_{\text{FRW}}} = \frac{NA}{4\pi r_{\text{obs}}^2 a(t_{\text{obs}})^2} \frac{4\pi r_{\text{FRW}}^2 a(t_{\text{obs}})^2}{NA} \quad (15)$$

$$= \frac{r_{\text{FRW}}^2}{r_{\text{obs}}^2} \geq 1. \quad (16)$$

Telescopes in a perturbed FRW universe receive on average more photons from a source at a given redshift than telescopes with the same area in a FRW universe and therefore have a higher apparent magnitude. This is the main result of this paper.

#### 4. DISCUSSION

The focusing theorem (Schneider et al. 1992, page 132) shows that a light beam is magnified if it is affected by gravitational lensing but does not go through a caustic. In light of the focusing theorem, the result in Section 3 is not surprising. The conclusion of Section 3 is significant because it shows that gravitational lensing can cause magnification for all observers without violating conservation of photon number.

The calculation in Section 3 is very general in that it does not depend up the distribution of matter, only that there are matter perturbations. It does assume that the global structure of the universe is FRW and that the departure from FRW is slight. It seems plausible that any greater departure from FRW will not remove the effect. Furthermore, the argument is based on the non-linearity of the relationship between matter and the metric, so it is easy to see how it may be applied in models other than FRW.

I am grateful to L. Ryder, J. Adams, W. Joyce, S. Seunarine and S. Besier for discussions and careful reading of drafts.

#### REFERENCES

- Claudel, C. 2000, Proc. R. Soc. London Ser. A, 465, 1455  
 Ellis, G. F. R., Bassett, B. A. C. C., & Dunsby, P. K. S. 1998, Class. Quantum Grav., 15, 2345  
 Peebles, P. 1993, Principles of Physical Cosmology, Princeton Series in Physics (Princeton New Jersey: Princeton University Press)  
 Perlmutter, S., et al. 1998, ApJ, 517, 565  
 Schmidt, B. P., et al. 1998, ApJ, 507, 46  
 Schneider, P., Ehlers, J., & Falco, E. 1992, Gravitational Lenses, A&A library (New York: Springer-Verlag)  
 Weinberg, S. 1976, ApJ, 208, L1